Abstract
This article presents a conceptual framework for operationalizing strategic enterprise risk management (ERM) in a general firm. We employ a risk-constrained optimization approach to study the capital allocation decisions under ERM. Given the decision maker's risk appetite, the problem of holistically managing enterprise-wide hazard, financial, operational, and real project risks is treated by maximizing the expected total return on capital, while trading off risks simultaneously in Value-at-Risk type of constraints. This approach explicitly quantifies the concepts of risk appetite and risk prioritization in light of the firm's default and financial distress avoidance reflected in its target credit rating. Our framework also allows the firm to consider a multiperiod planning horizon so that changing business environments can be accounted for. We illustrate the implementation of the framework through a numerical example. As an initial conceptual advancement, our formulation is capable of facilitating more general ERM modeling within a consistent strategic framework, where idiosyncratic variations of firms and different modeling assumptions can be accommodated. Managerial implications are also discussed.

Motivation for the Research
Enterprise risk management (ERM), considered a breakthrough idea by the Harvard Business Review (Buchanan, 2004), emphasizes that all risks should be treated holistically from an enterprise-wide perspective (as opposed to individually in “silos”). This concept has been widely embraced by industry, regulatory forces (e.g., Sarbanes Oxley Act of 2002, Basel Capital Accord II), rating agencies (e.g., Standard & Poor’s, 

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This article offers a mathematical approach to operationalizing the integration of ERM within the firm to achieve its holistic strategic goals across time periods. The overall purpose of this article is to provide both a conceptual framework and mathematical tools to make the implementation of ERM more concrete. There are significant challenges to the implementation of ERM (Gate, 2006) as risk considerations have yet to be fully integrated into business decision making (Deloitte, 2008). We attempt to provide an approach to overcoming some of these challenges.

In order to do this, we first have to adopt a perspective on what ERM is as it is still an emerging and evolving concept. The prevailing formulation of ERM that we adopt as our perspective was offered by COSO (2004). This formulation includes the crucial concept of risk appetite: risk appetite is a corporation’s willingness and ability to undertake risks to achieve its strategic objectives that governs business decision making. It also recognizes interrelations between risks and the prioritization of risks (i.e., rank order of risk types according to importance), which is critical to holistic integration, a goal of our mathematical approach. This holistic integration is an important characteristic of the stated end-goal for ERM: to gain competitive advantage and create value (Economic Intelligence Unit, 2007).

ERM Decision Framework
The Setting

We present our ERM framework in a general setting to incorporate fundamental aspects of ERM. For simplicity, we consider a two-stage dynamic risk/return optimization problem with a two-period planning horizon, wherein the firm plans for the entire horizon (i.e., both periods) in stage 1 and can later change period 2 decisions upon receiving new information in stage 2. This dynamic formulation allows the firm to adapt to changing internal and external environments, which is one of the key drivers for ERM development (Protiviti, 2007). The proposed ERM framework allows for managerial inputs (e.g., risk appetite, risk limits, and risk prioritization) at the onset.

Under our framework, the firm’s strategic goals are encompassed within an objective function, and risk concerns are controlled in impact and likelihood by using

\[ \text{No single definition or perspective on ERM is accepted by everyone. The specifics of what ERM is remains controversial. We adopted the COSO (2004) perspective on what ERM is because it can be described reasonably as the prevailing one. Other current models, however, define ERM in terms of risk aggregation, which is important for determining the amount of economic capital needed for financial institutions (e.g., Rosenberg and Schuermann, 2006). While risk aggregation perspective models capture interrelationships between risks, they do not explicitly address risk appetite, prioritization, operational decisions, trade-offs among different risk categories, etc., making these models less applicable to a general industrial firm’s business decision making.} \]

\[ \text{We present our framework as a two-stage model; however, it can be extended to a multistage model for a multiperiod planning horizon (see the “Extension to a Multistage Model” section for more discussion).} \]
constraints. We address two broad types of investment opportunities: short-term investment that can be reassessed each period or long-term investment that cannot be changed for the entire two-period planning horizon. These investment opportunities can include real manufacturing projects (e.g., production projects for a manufacturing firm or lines of insurance for an insurance company) and liquid or illiquid financial assets. In addition to uncertainty in project and financial investment returns, the firm also faces operational and hazard risks (fire, theft, etc.). By convention, hazard risks are managed by the selection of insurance coverage levels. Operational risks depend upon the duration and the nature of the investment.

Model Notations. Our dynamic model allows decision makers the option to invest capital at the beginning of period 2 based on newly arriving information, so we provide a distinct notation for the investment returns, decision variables, and other parameters based on information at the beginning of period 1 (time 0) and the beginning of period 2 (time 1).

1. We let subscripts $S$ and $L$ denote, respectively, a one-period (or short-term) investment and a two-period (or long-term) investment, with additional subscripts added to designate multiple investment opportunities when necessary and superscripts to indicate the time period.

2. We let $\mathcal{F}_0$ and $\mathcal{F}_1$ denote information sigma-algebras of information available at time 0 and time 1. Let $R^{(i)}_S$ denote total return (i.e., $1 + \text{rate of return}$) on a short-term investment in period $i$ ($i = 1, 2$), and $r^{(i)}_S = E[R^{(i)}_S | \mathcal{F}_0]$ denote the (stochastic) best assessment of return during period $i$ given information at time 0. Similarly, $r_L = E[R_L | \mathcal{F}_0]$ denotes total return on a long-term investment over two periods as assessed at time 0; that is, the long-term investment consumes resources during period 1 and returns $r_L$ at the end of period 2.

3. At time 1, returns of the short-term investments in period 1 are realized, and new strategic information that has arisen is incorporated into the new information sigma-algebra $\mathcal{F}_1$. At the onset of period 2, we let $\bar{r}^{(2)}_S = E[R^{(2)}_S | \mathcal{F}_1]$ denote revised (stochastic) assessment of returns on short-term investments in period 2 given information at time 1. Due to new environmental or market information, $\bar{r}^{(2)}_S$ may not equal $r^{(2)}_S$, so strategic decisions may evolve. Similarly, we define $\bar{r}_L = E[R_L | \mathcal{F}_0]$ for long-term investments.

4. Let decision variables $w^{(1)}_S$ and $w^{(2)}_S$ denote proportions of capital planned at time 0 to be committed to short-term investments in period 1 and period 2, respectively and $w_L$ is the proportion committed to long-term investment at time 0. Here $w^{(2)}_S$
defines a strategic plan at time 0 intended to be implemented for period 2 but that may change at the end of period 1 in light of new information.

(5) At time 1 (stage 2 of the model), the firm updates period 2 decisions. Let $\bar{w}^{(2)}_S$ be updated decisions for proportions of capital committed to short-term investments in period 2 given $\mathcal{F}_i$.

(6) The firm has capital $C^{(0)}$ and $C^{(1)}$ to allocate at time 0 and time 1, respectively. Capital available at time 1, $C^{(1)}$, depends on realizations of returns and losses during period 1, so at time 0 $C^{(0)}$ is known and $C^{(1)}$ is stochastic (denoted as $\tilde{C}^{(1)}$), but $C^{(1)}$ becomes known for the stage 2 optimization.

(7) At time 2 (end of period 2), the firm has capital $C^{(2)}$. The goal of stage $i$ ($i = 1, 2$) optimization is to maximize $E[\tilde{C}^{(2)}|\mathcal{F}_{i-1}]$ subject to period $i$ constraints using conditional probabilities given $\mathcal{F}_{i-1}$.

Model Assumptions. We consider four categories of risks: project risk, financial risk, operational risk, and physical hazard risk. Financial and project risks are speculative, whereas operational and hazard risks are pure risks. Conventional risk management for hazard risk involves insurance arrangements, so we incorporate the level of insurance coverage as a decision variable. To facilitate this, we make three assumptions on hazard risk.

(1) We measure the direct financial impact of hazard risk (loss of capital due to natural hazards and liability) as a proportion of the firm’s capital $C$. Indirect loss caused by hazard risk (such as income loss from business interruptions) is assumed to be proportional to direct hazard loss. Let the unit hazard loss (both direct and indirect losses) be characterized as a random variable $h$ (and thus total hazard loss is $hC$). More specifically, based on $\mathcal{F}_0$, unit hazard risk in period 1 and period 2 is, respectively, $h^{(1)}$ and $h^{(2)}$ with expected value $\mu^{(1)}$ and $\mu^{(2)}$. At time 1 based on $\mathcal{F}_0$, unit hazard risk in period 2 is assessed as $\bar{h}^{(2)}$ with expected value $\bar{\mu}^{(2)}$.

(2) Insurance premiums include an additional loading factor $d$ (for the insurance company’s profit, expenses, etc.) on expected losses. At time 0, the firm is charged insurance loading $d^{(1)}$ for period 1 and we assume there will be projected loading in period 2, $d^{(2)}$. At time 1, the firm is actually charged $\bar{d}^{(2)}$.

(3) At time 0, the firm plans to insure a percentage $u^{(i)}$ of hazard risk for period $i$ ($i = 1, 2$). At time 1, the firm updates decisions and insurers a proportion $\bar{u}^{(2)}$ in period 2.

The Objective Function

The strategic objective we consider is to maximize the total expected end-of-horizon wealth through investment in real projects and financial assets along with consideration for insurance purchase. The total return will be in the form $\Pi = C wr$, where $w$ and $r$ are vectors of appropriate dimensions if multiple investment opportunities are considered. The insurance premium expense is deducted directly from the total expected return, and thus, we obtain the general form of objective function
The second term reflects extra expenses incurred due to insurance loading. However, insurance purchase \((u > 0)\) can be justified in a constrained model since it may help the firm satisfy risk considerations. As shown below, the specifications of the objective function for each stage are similar conceptually but different notationally.

**The First-Stage Objective Function.** At time 0, the firm plans operational decisions for the entire planning horizon based on information \(\mathcal{F}_0\). Decisions for period 1 will be implemented immediately while decisions for period 2 are strategic plans and can change at time 1 in light of new information. The strategic goal is to maximize expected returns over the entire planning horizon, that is,

\[
\max_{w_1, u_1, u_2} \mathcal{C}(0) E \left\{ \left[ w_1^{(1)} r_1^{(1)} - u_1^{(1)} d_1^{(1)} \mu^{(1)} \right] \times \left[ w_1^{(2)} r_1^{(2)} - u_1^{(2)} d_1^{(2)} \mu^{(2)} \right] + w_1 L r_L \right\}.
\]

**The Second-Stage Objective Function.** At time 1, the firm receives new information set \(\mathcal{F}_1\). Here the superscript * indicates the value obtained for the investment decisions as a result of stage 1 optimization, and \(\wedge\) indicates realizations of the period 1 random variables now known at time 1. In stage 2 optimization, the firm is not required to stick with the original investment and risk management plans for period 2 \((w_1^*, u_1^*)\) if new information leads to other more profitable choices. The objective is still to maximize end-of-horizon (time 2) total expected return, that is,

\[
\max_{w_1^*, u_1^*, w_2, u_2} \mathcal{C}(0) E \left\{ \left[ w_1^{* (1)} r_1^{* (1)} - u_1^{* (1)} d_1^{* (1)} \mu^{(1)} \right] \times \left[ \bar{w}_2^{(2)} r_2^{(2)} - \bar{u}_2^{(2)} d_2^{(2)} \bar{\mu}_2^{(2)} \right] + w_1^* L r_L \right\}.
\]

**The Constraints**

While the strategic goal of maximizing end-of-horizon wealth is specified in the objective function, the constraints address risks and other considerations in pursuit of this goal. Two types of constraints are used: stochastic constraints similar to Value-at-Risk (VaR) measure and deterministic constraints.

We use VaR measure of risk because it is a very commonly used method for specifying risk in financial applications. In fact, according to Tiesset and Troussard (2005), “Although the methodologies for measuring risk are still evolving, Value-at-Risk (VaR) is the most widely used method for estimating aggregate risk” (p. 63). It is also a risk measure often required by regulatory agencies (e.g., cf. Basel II, 2004). VaR has been shown to be a coherent risk measure for elliptical multivariate distributions (Embrechts, McNeil, and Straumann, 2002). In addition, we use VaR because it allows

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\[\text{At this point the firm could alternatively choose to do another two-period projection of allocations involving new assessments of investment opportunities, risks, etc., as done in stage 1, with other potential two-period projects commencing at time 1 and continuing to time 3.}\]
us to address in a single measure the topics of risk appetite(s) and risk prioritization that are fundamental to ERM, as detailed below in the “Stochastic Constraints for Period 1 (in Stage 1)” section. An overview of quantile-based risk measures in the finance and insurance context, including VaR, is given in Dowd and Blake (2006).

We allow each of the four risk types (i.e., project, financial, operational, and hazard risk) to be considered in a separate constraint to provide for different risk appetites and risk limits for each risk. We capture prioritization among risks (such as determined by traditional risk maps) by the relative magnitudes of individual risk appetites. Managerial expertise and preference are utilized at this juncture. We also add a solvency constraint encompassing all risk types to comply with the firm’s overall risk appetite that governs all business activities. Other considerations (e.g., budgetary or regulatory considerations) are captured in deterministic constraints.

We discuss in detail the formulation of each constraint for period 1 in stage 1, and summarize the constraints for period 2 in each stage since they have similar structures.

Stochastic Constraints for Period 1 (in Stage 1). In the interest of making the ERM optimization more explicit, we shall henceforth consider two types of short-term investments: real projects that occupy a single period and liquid financial investments. We also consider long-term real projects (such as research and development) as well as illiquid financial investments that consume resources at the beginning of period 1 but do not pay off until the end of period 2. In the vector notation used in the objective function, we partition \( w^{(1)} \) into \((w_{SP}^{(1)}, w_{SA}^{(1)})\) for short-term real projects and liquid financial assets, respectively. Similarly, \( w_L \) is partitioned into \((w_{LP}, w_{LA})\), and similar partitions are used for the return vectors.

**Project risk:** Project risk refers to possible unfavorable outcomes from real project investments. “Project” is broadly defined to include all activities associated with the real business part of the firm. To control project risk, the firm selects projects so that a prespecified minimum level of total return (risk limit or hurdle rate) can be secured with specified confidence. This is governed by the firm’s risk appetite toward project risk, which is a managerial input concerning other firm characteristics (e.g., the credit rating target of the firm). Risk appetite for project risk is denoted by \( \alpha_1 \). Discussion related to the managerial selection of parameters such as \( \alpha_1 \) is given in the “Choice of Risk Appetite and Other Parameters” section following the model formulation.

Let \( r_{P0}^{(1)} \) be the (deterministic) minimum total return desired per unit of capital invested in real projects. Mathematically, we formulate the project risk constraint as

\[
P\left[ C^{(0)}w_{SP}^{(1)}r_{SP}^{(1)} \leq C^{(0)}\left(w_{SP}^{(1)}e + w_{LP}e\right)r_{P0}^{(1)} \right] \leq \alpha_1, \quad (\text{Cons 1-1})
\]

where \( e \) is a vector of 1’s of appropriate dimension.

To satisfy (Cons 1-1), the firm must balance investments in different short-term and long-term projects to ensure that project risk in period 1 is within the firm’s project risk appetite \( \alpha_1 \).
**Financial risk:** Financial risk is prototypically risk from financial investment and can have characteristics different from real projects (e.g., tradability, liquidity, and hedging methods). In ERM, hedging opportunities may already exist among financial assets, between financial assets and real projects, or among a broader range of business components/units even before any targeted risk management, that is, so-called “natural hedges.” Our model takes advantage of these natural hedge opportunities by allowing different business component activities to interact through constraints and the dependence structure.

Other than natural hedges, we do not impose a separate hedging policy for financial risk, so the constraint on financial risk is formulated similar to the project risk constraint. The firm may have a different risk appetite ($\alpha_2$) for financial risk. Specifically, the financial risk constraint is

$$P \left[ C^{(0)} w^{(1)}_{SA} r^{(1)}_{SA} \leq C^{(0)} \left( w^{(1)}_{SA} e + w^{(1)}_{LA} e \right) r^{(1)}_{A0} \right] \leq \alpha_2,$$

(Cons 1-2)

where $r^{(1)}_{A0}$ is the minimum required return per unit of capital invested in the portfolio of financial assets.

**Operational risk:** Operational risk is a nascent risk category recently ranked as the most important risk domain by U.S. corporate executives (Towers Perrin, 2006). It spans a variety of internal and external business activities (cf. Basel Committee, 2004). Currently there are few data available even at the industry level for most industries (cf. Guillén et al., 2007). Unlike hazard risk and financial risk, there is not yet any commonly accepted approach to quantify operational risk or to set the risk constraint accordingly.

To construct a stochastic constraint on operational risk, we follow the spirit of the Standardized Approach from the Basel Capital Accord II (Basel Committee, 2004) wherein the operational risk corresponding to a business component of the firm is a multiple of the underlying risk exposure in this component. We use the Standardized Approach from Basel II because it is a well-known uniformly applicable approach and no other currently available approach will provide such general applicability. Following this approach, a proportion, $\gamma_P \geq 0$, of total project return is estimated to be attributable to operational risk for projects. Similarly, a proportion, $\gamma_A \geq 0$, corresponds

5 A hedging policy could be incorporated in our framework should the firm wish to hedge financial risk (cf. Caldentey and Haugh, 2006). Similar to the way we address hazard risk in our model, one could introduce a hedge ratio parameter for financial risk as a decision variable and incorporate it into the objective function. The stochastic constraints then involve “after-hedging returns” required to exceed the hurdle rate with the specified probability (1-risk appetite). We do not consider this extension here but instead assume that the returns on assets are “after hedging” or without hedging.

6 Basel II proposed three approaches: Basic Indicator Approach, Standardized Approach, and Advanced Measurement Approach (AMA). AMA is more sophisticated but requires estimating loss distributions from operational loss databases, which are currently available only for a few particular industries (like financial services) at best (e.g., Guillén et al., 2007). Our formulation is to be generally applicable, so the Standardized Approach is more useful.
to operational risk in financial investments. Applying these factors to respective
total returns yields an estimate of operational risk from projects and financial asset
investments. The $\gamma$s could differ to partly reflect different relative risk exposure of
various business components. Note that the firm can also similarly impose different
$\gamma$s on different subclasses of short-term and long-term investments.

While returns from long-term projects are not realized until period 2, they are subject
to operational risk in both periods. Thus, we use the total return from the long-term
projects $w_{LR_{1}}$ to measure the time 0 assessment of the operational risk exposure
for long-term projects in period 1 (and in period 2). Accordingly, the constraint for
operational risk in period 1 is

$$
P \left[ \gamma_{P}^{(1)} C^{(0)} \left( w_{SP_{1}}^{(1)} r_{SP_{1}} + w_{LP_{1}} r_{LP_{1}} \right) + \gamma_{A}^{(1)} C^{(0)} \left( w_{SA_{1}}^{(1)} r_{SA_{1}} + w_{LA_{1}} r_{LA_{1}} \right) \right] \geq C^{(0)} l_{op_{1}} \right] \leq \alpha_{3}, \quad \text{(Cons 1-3)}
$$

where $\gamma_{P}^{(1)}$ and $\gamma_{A}^{(1)}$ are, respectively, the operational risk factors for projects and fi-
nancial assets and $l_{op_{1}}$ is the specified risk limit. Thus, (Cons 1-3) requires that the
likelihood of exceeding a given operational risk limit to be restricted within the firm’s
operational risk appetite $\alpha_{3}$.

**Hazard risk:** The firm insures a proportion $u^{(1)}$ of hazard risk in period 1. Assume
that the firm is willing to retain a maximum hazard loss of $l_{h_{1}}$ per unit of capital (risk
limit), subject to risk appetite $\alpha_{4}$. The hazard risk constraint is formulated as

$$
P \left[ \left( 1 - u^{(1)} \right) h^{(1)} C^{(0)} \geq l_{h_{1}} C^{(0)} \right] \leq \alpha_{4}. \quad \text{(Cons 1-4)}
$$

**Overall risk:** In addition to individual risk-type constraints, we impose a solvency
constraint based on the firm’s overall risk appetite $\alpha_{5}$. This constraint requires that
returns in period 1, after any losses, are sufficient to repay any financial obligation
$c^{(0)}$ due at the end of period 1 (debt or fixed financial obligations including rent, etc.)
with probability $(1 - \alpha_{5})$

$$
P \left[ \left( 1 - \gamma_{P}^{(1)} \right) C^{(0)} w_{SP_{1}}^{(1)} r_{SP_{1}} - \gamma_{P}^{(1)} C^{(0)} w_{LP_{1}} r_{LP_{1}} + \left( 1 - \gamma_{A}^{(1)} \right) C^{(0)} w_{SA_{1}}^{(1)} r_{SA_{1}} - \gamma_{A}^{(1)} C^{(0)} w_{LA_{1}} r_{LA_{1}} - (1 - u^{(1)}) h^{(1)} C^{(0)} - u^{(1)} (1 + d^{(1)}) \mu^{(1)} C^{(0)} \right] \leq c^{(0)} \right] \leq \alpha_{5}. \quad \text{(Cons 1-5)}
$$

The parameter $c^{(0)}$ is exogenous and known to the firm at time 0.

*Deterministic Constraints for Period 1 (in Stage 1).* A few deterministic constraints are
in place to capture regulatory and other considerations.
Budget constraint: Total capital in period 1 is allocated to invest in projects and financial assets and to pay for the insurance premium for hazard risk. Thus, the budget constraint is

\[ w_{SP}^{(1)}e + w_{LP}^{(1)}e + w_{SA}^{(1)}e + w_{LA}^{(1)}e + u^{(1)}(1 + d^{(1)})\mu^{(1)} \leq 1. \]  
(Cons 1-6)

Strategic/regulatory constraint: It may be desirable to have a minimum amount of capital invested in real projects for strategic purposes in each period (e.g., to keep market presence, to keep current employment staff in force for subsequent time periods, etc.) or to comply with regulatory requirements (e.g., on maximum investment in financial assets for industries such as the insurance industry). The associated constraint is then

\[ w_{SP}^{(1)}e + w_{LP}^{(1)}e \geq \gamma_S^{(1)}, \]  
(Cons 1-7)

where \( \gamma_S^{(1)} \) is a prespecified minimum percentage invested in real projects.

Range constraints:

\[ w_{SP}^{(1)} \geq 0, \ w_{SA}^{(1)} \geq 0, \ w_{LP} \geq 0, \ w_{LA} \geq 0, \ 0 \leq u^{(1)} \leq 1. \]  
(Cons 1-8)

Stochastic Constraints and Deterministic Constraints for Period 2 in Each Stage. The constraints for period 2 in stage 1 and stage 2 are constructed similar to those detailed for period 1 but with different capital to allocate to different investment opportunities. In stage 1, the firm plans the allocation of the expected capital for period 2 \( (E[C^{(1)}|I_0]) \) with investment return characteristics assessed based on the information set \( I_0 \). In stage 2, the known amount of capital \( C^{(1)} \) (determined by realized returns and implemented optimal decisions in period 1) is allocated based on information set \( I_1 \). Notations are changed to reflect the different time period and/or updated information. More specifically, (Cons 1-1) to (Cons 1-8) have period 2 analogues (Cons 2-1) to (Cons 2-8) using the information set \( I_0 \), with time period superscript “(2)” replacing superscript “(1).” Note also that the long-term investment decisions made in period 1 are now generating returns in period 2. The second-stage optimization for period 2 have analogues (Cons 2'-1) to (Cons 2'-8) using the information set \( I_1 \), with a bar symbol (\( \bar{\ } \)) indicating updated decisions, return assessments, and other parameters. These period 2 constraints (for each stage) are explicitly given in the Appendix.

The Optimizing ERM Framework

Our two-stage ERM decision framework optimizes the firm’s global strategic goal of maximizing end-of-horizon expected returns in the objective function in each stage while addressing risk considerations in the constraints for each period in each stage. Our formulation is summarized as below.
The First Stage

\[
\text{Max} \quad w_1^{(1)}, w_2^{(2)}, w_1^{(1)}, u^{(2)} \\
\text{s.t. Period 1 constraints: (Cons 1-1) to (Cons 1-8)} \\
\text{Period 2 constraints: (Cons 2-1) to (Cons 2-8)}
\]

The Second Stage

\[
\text{Max} \quad \bar{w}_2^{(2)}, \bar{u}^{(2)} \\
\text{s.t. Period 2 constraints: (Cons 2′-1) to (Cons 2′-8)}
\]

Discussions and Extensions of the Dynamic ERM Framework

Distributional Assumptions of the Model and Solution Techniques. Since the VaR type constraints that arise in our ERM optimization involve linear combinations of random variables within the probability operator, the distribution of such linear combinations is important in order to calculate the constraint. In general, if we know the distribution function \( F \) of the random variable \( X \) within the probability operator of the constraint, a constraint like \( P [X \leq r] \leq \alpha \) can be expressed equivalently as \( r \leq F^{-1}(\alpha) \), a deterministic constraint, making the optimization a linear objective function subject to deterministic constraints. The constraint set will generally form a compact set (often convex) allowing for computation of the optimal values (cf. Mas-Colell, Whinston, and Green, 2005). The problem, of course, is that \( X \) is a linear combination of random variables for which we may know marginal distributions, but the joint distribution of the components that the combination \( X \) depends on may not be known in closed form.

In a very general setting, each individual risk can be expressed in terms of its marginal distribution and copula methods (cf. Embrechts, McNeil, and Straumann, 2002) can be used to model the multivariate dependence structure to construct the joint distribution. The probabilistic constraints appearing in our ERM formulation could then be calculated from the copula. For example, Rosenberg and Schuermann (2006) use copula methods to aggregate risks for VaR computations in a financial institution portfolio context.

For certain joint distributions, such as multivariate normal, elliptically symmetric, or skewed distributions, the computation can be carried out by developing the “deterministic equivalents” (cf. Cooper, Lelas, and Sullivan, 2006, who also address a method for performing computations in spite of nonconvexity). Computations in more general situations involving log-concave distributions is shown by Prekopa (1971) to determine a convex constraint set and standard techniques can be employed. In the most general situation, for the formulation developed in this article,

\[\text{7Please refer to the sections “The Objective Function” and “The Constraints,” and the Appendix for the specifications of objective functions and constraints.}\]
once an appropriate joint distribution has been created (by copulas or other methods) a genetic algorithm can be used to derive the solution (cf. Poojari and Varghese, 2008). The availability of these techniques makes sure that our model is implementable under different distributional assumptions driven by practical considerations. It also allows for variations and extensions to our current framework.

Incorporation of Risk Management Strategies. Aside from hazard risk, we do not impose specific risk management strategies in our current formulation, leaving it open to various risk management choices by the firm. The selected risk management activities will normally alter the risk distributions in the constraints (e.g., truncate the range of possible values) and entailed costs can be included in the objective function as was done for hazard risk. In addition, the effect of risk management may be reflected in the parameterization as well, for example, proper risk management activities can result in lower factor loadings ($\gamma_P$ and $\gamma_A$) for operational risk.

Extension to a Multistage Model. The model herein is a two-stage model, chosen to keep the presentation simple. However, it can be extended to a multistage model following the same rationale. In an $N$-stage model, the firm has an $N$-period planning horizon. In stage 1 at time 0, planning decisions are made for each period in the horizon, based on information $\mathcal{I}_0$. Decisions for period 1 are implemented immediately. In stage 2, new information, $\mathcal{I}_1$, becomes available, including realizations from period 1. The firm then updates and implements its decisions for period 2 based on $\mathcal{I}_1$. More generally, in stage $n$ ($n = 2, \ldots, N$), the firm uses information $\mathcal{I}_{n-1}$ to update and implement its decisions for period $n$. Here the long-term investments can last 2 to $N$ periods. For an $n$-period long-term investment, capital is committed at the beginning of period $i$ ($i = 1, \ldots, N - n + 1$) and return is not realized until the project is finished. At the end of period $N$, all returns are realized and all optimized decisions are implemented. Risk constraints for each period of the planning horizon in each stage are constructed similarly as in the two-stage model.

Other extensions can be accommodated such as allowing additional injections of capital for the long-term investment during its duration by deducting them in each period from that period’s total capital and making an adjustment to the constraints. The model can also be similarly adjusted to allow long-term investments to generate returns in multiple periods.

**CHOICE OF RISK APPETITE AND OTHER PARAMETERS**

We use two types of parameters in our ERM framework: exogenous parameters and decision parameters. Exogenous parameters are given to the decision makers (e.g., investment return parameters). Decision parameters, on the other hand, are within managerial discretion (e.g., risk appetite parameters) and are used as inputs to the decision making optimization model. We now give guidance on the managerial choice of decision parameters.

**Risk Appetite**

Recognition and incorporation of risk appetite into the strategic decision process is an important feature of ERM. For this process, a firm articulates the choice of risk appetite parameters in line with its strategic goals and risk culture. Previously
these parameters were taken as given and incorporated in stochastic constraints. We now elaborate on how one might determine quantitatively firm-specific risk appetite parameters.

**Overall Risk Appetite.** The choice of overall risk appetite is guided by a major performance target of the firm, namely, the credit rating (cf. Nocco and Stulz, 2006). Credit rating determines a firm’s ability to raise capital and the associated cost of capital (West, 1973), and can affect contract provisions with other constituencies. Credit rating concerns also influence corporate policies and business strategies such as capital investment (Sufi, 2009). Thus it can serve as a proxy for the strategic goal governing risk appetite(s). In addition, the target credit rating is a readily identified parameter and can be a starting point for analysis.

Specifically, target credit rating translates into an implied target probability of default and a target probability of financial distress. To maintain its target credit rating, a firm should restrict the default probability to be within the level implied by that credit rating, i.e., the solvency constraint in our model. This level of default probability defines the overall risk appetite, $\alpha_5$. This establishment adopts the solvency concern that Standard & Poor’s identifies as one of the main themes for risk appetite determination in their rating process (Standard & Poor’s, 2006b).

**Risk Appetite for Individual Risk Types.** In our ERM operationalization, we have standardized the description of risk appetites among individual risk types through the use of parameters $\alpha_i$s. This allows us to capture risk prioritization by the relative magnitude (i.e., rank order) of individual risk appetite parameters ($\alpha_i$s) in accordance with a priori managerial prioritization (e.g., obtained by traditional ERM risk maps). Consistent with the “target credit rating” theme for the overall risk appetite, one way to obtain the exact values is to allow $\alpha_i$s to vary around the probability of financial distress implied by the target credit rating. The rationale is that each risk puts pressure on the financial soundness of the firm, so even for less prioritized risk types (i.e., ones the firm is more willing to take), the firm may want to limit its impact to avoid financial distress to the extent of the target probability. While the credit rating target provides a reasonable general guideline for risk appetite articulation, other themes can be used as well to determine these parameters depending on the firm’s overall strategic goals (cf. AIRMIC, 2009).

**Other Parameters**

The ERM framework involves other parameters supplied by the firm, including risk limits ($r_{P0}, r_{A0}, l_{op}, l_{h}$), operational risk loading factors ($\gamma_P, \gamma_A$), and minimum real project investment requirement $\gamma_S$. Risk limits can be chosen correspondingly to reflect each risk’s impact on financial distress, determined using firm-specific historical data, industry-level data, or managerial assessment. Risk limits may also come from

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8Default probabilities associated with various credit ratings can be obtained using, for example, the rating transition matrix in Moody’s default and recovery rates of corporate bond issuers: 1920–2005, March 2006. Financial distress probability can also be similarly derived if using bond rating to determine the financial distress threshold (cf. Nocco and Stulz, 2006).
regulatory requirements or benchmarking, especially for financial firms (cf. Basak, Shapiro, and Teplá, 2006).

The operational risk loading factors should be based on quantitative analysis using firm historical data (or industry data if available) and/or qualitative analysis of each business component’s exposure to operational risk. The minimal proportion of capital invested in real projects may be based on a firm’s overall strategies and balance between different constituencies, or based on regulatory requirements. Finally, there are additional range constraints: $0 \leq \alpha_i \leq 0.5, i = 1, \ldots, 5,$ and other nonnegativity parameter constraints.

**NUMERICAL ILLUSTRATION**

We now give a numerical illustration, showcasing the choice of baseline parameters and the implementation of our ERM decision framework. We examine the firm’s decisions in both stages and exhibit the dynamic nature of the model (i.e., how the firm’s time 0 assessment for period 2 can affect its decisions for period 1). Since firms can have different strategies and risk culture leading to varied choice of parameters, we also use a set of prototype analyses to examine how firms can incorporate these different considerations and explore the implications.

**Distributional Assumptions Used in the Numerical Illustration**

As discussed previously in the “Distributional Assumptions of the Model and Solution Techniques” section, the model proposed in this article can accommodate numerous distributional assumptions. The most efficient computational techniques, however, may differ depending upon the form of the distribution used. In this example we use the multivariate normal formulation because of its computational ease, as the purpose of the illustration is to demonstrate the implementation of our ERM framework. We do not want to distract from this purpose with lengthy presentation of the computations necessary for various other distributions (see, e.g., Prekopa, 1971; Cooper, Lelas, and Sullivan, 2006; Poojari and Varghese, 2008, for efficient computations under alternative distributional assumptions).

(1) The returns on all projects and financial assets assessed at time 0 ($r_{S}^{(1)}, r_{S}^{(2)}, r_{L}$) follow a multivariate normal distribution, with the vector of expected returns ($E(r_{S})^{(1)} : E(r_{S})^{(2)} : E(r_{L})$) and a positive semi-definite variance–covariance matrix $\Sigma^{(r)}$. Similarly, the returns on projects and financial assets assessed at time 1 ($\bar{r}_{S}^{(2)}, \bar{r}_{L}$) are random variables assumed to follow a multivariate normal distribution, with the vector of expected returns ($E(\bar{r}_{S})^{(2)} : E(\bar{r}_{L})$) and a positive semi-definite variance–covariance matrix $\Sigma^{(\bar{r})}$.

Note that different distributional parameters are allowed for time 0 and time 1 assessment of period 2 investment returns, since new information may have arrived during period 1 for the firm to use in the assessment. Note also that the use of the total covariance matrix $\Sigma^{(r)}$ and $\Sigma^{(\bar{r})}$ allows for correlations across investments and operational risks. It does not treat these risks individually as “silos,” but rather allows for the possibility of natural hedges between these risk types as is desirable in ERM strategies.
(2) We assume time 0 assessment of hazard risk per unit capital for period \( i (i = 1, 2) \), \( h^{(i)} \), follows a Normal distribution, \( h^{(i)} \sim N(\mu^{(i)}, \sigma^{(i)}^2) \), \( \mu^{(i)} > 0, \sigma^{(i)} > 0 \). The time 1 assessment for unit hazard risk in period 2 is \( \bar{h}^{(2)} \), where \( \bar{h}^{(2)} \sim N(\bar{\mu}^{(2)}, \bar{\sigma}^{(2)}^2) \). We assume that hazard risk is independent of project and asset returns since accidental or “acts of God” losses are usually uncorrelated with investment returns.

A Numerical Illustration for the Dynamic Model

In our numerical illustration, the problem is to make ERM-oriented optimal operational decisions for a 2-year planning horizon. Since we do not have access to a specific firm’s proprietary data, we use both real market-level data and simulated firm-level data to implement our model. We additionally investigate possible alternative parameterizations in the “Comparative Impacts of Different Parameterizations” section, as these may impact risk management and operational decisions.

For specificity, we consider an investment portfolio with three real projects (two 1-year short-term projects, and one 2-year long-term project) and two short-term financial assets. The financial assets are an index fund (S&P Depositary Receipts) that tracks the S&P 500 index and the 3-month Treasury bill. We use available historical data (02:1993 – 12:2006) to obtain expected returns and variances for 1-year holding period resulting in an estimated expected return and variance of 12 percent and 3 percent for the index fund, and 3.8 percent and 0.025 percent for the Treasury bill.

The three real projects considered have the following different risk/return characteristics: a risky long-term research and development (R&D) strategic project (expected return 60 percent, variance 15 percent), a short-term routine manufacturing project no. 1 (expected return 8 percent, variance 0.1 percent), and another short-term adventurous manufacturing project no. 2 (expected return 20 percent, variance 4 percent). A small negative correlation (cf. Breen, Glosten, and Jagannathan, 1989) is imposed between the financial assets, and a small positive correlation is imposed among the real projects. Correlations between the index fund and the projects are positive, while correlations between the Treasury bill and projects are negative. We impose higher correlation within the investment classes than across the investment classes. In our baseline model presented in this subsection, we assume that time 0 assessments of returns in period 1 and period 2 are the same for short-term investments.

As per the model, the firm updates assessments of period 2 investments at time 1 based on newly available information. For the numerical illustration, we assume that at time 1 it is assessed that the economy is in recession and the return prospects are revised accordingly. Namely, in stage 2 of the model, expected returns for real projects and the index fund are reduced, volatilities of all returns are increased, and there is a higher correlation between the index fund and the Treasury bill. Table 1 presents time 0 and time 1 (in parentheses) assessments of investment opportunities. Stage 2 optimization requires inputs from the realizations of random return variables in period 1, which, for illustration purposes, are taken to be the expected returns assessed at time 0.

Table 2 presents the other parameter values for the baseline setting. Decision parameters are not revised between periods in stage 1 since the firm generally uses separate
### Table 1
Time 0 Assessment of Investment Opportunities for Period 1 and Period 2, With the Subsequent Time 1 Assessment for Period 2 Given in Parentheses

#### Expected Return and Variance

<table>
<thead>
<tr>
<th></th>
<th>Manufacturing Project 1</th>
<th>Manufacturing Project 2</th>
<th>R&amp;D Project</th>
<th>Index Fund</th>
<th>3-Month T-Bill</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expected return</td>
<td>0.08 (0.06)</td>
<td>0.2 (0.18)</td>
<td>0.6 (0.3)</td>
<td>0.12 (0.08)</td>
<td>0.038 (0.04)</td>
</tr>
<tr>
<td>Variance</td>
<td>0.001 (0.002)</td>
<td>0.04 (0.06)</td>
<td>0.15 (0.2)</td>
<td>0.03 (0.05)</td>
<td>0.00025 (0.0003)</td>
</tr>
</tbody>
</table>

#### Correlation of the Investments

<table>
<thead>
<tr>
<th></th>
<th>Manufacturing Project 1</th>
<th>Manufacturing Project 2</th>
<th>R&amp;D Project</th>
<th>Index Fund</th>
<th>3-Month T-Bill</th>
</tr>
</thead>
<tbody>
<tr>
<td>Manufacturing project 1</td>
<td>1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.05</td>
<td>−0.05</td>
</tr>
<tr>
<td>Manufacturing project 2</td>
<td>1</td>
<td>0.08</td>
<td>0.05</td>
<td>−0.05</td>
<td>−0.05</td>
</tr>
<tr>
<td>R&amp;D project</td>
<td>0.05</td>
<td>1</td>
<td>0.05</td>
<td>−0.05</td>
<td>−0.05</td>
</tr>
<tr>
<td>Index fund</td>
<td>−0.1 (−0.2)</td>
<td>1</td>
<td>1</td>
<td>−0.1 (−0.2)</td>
<td>1</td>
</tr>
<tr>
<td>T-bill</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

#### Variance Covariance Matrix

<table>
<thead>
<tr>
<th></th>
<th>Manufacturing Project 1</th>
<th>Manufacturing Project 2</th>
<th>R&amp;D Project</th>
<th>Index Fund</th>
<th>3-Month T-Bill</th>
</tr>
</thead>
<tbody>
<tr>
<td>Manufacturing project 1</td>
<td>0.001 (0.002)</td>
<td>0.00632 (0.001095)</td>
<td>0.001225 (0.0002)</td>
<td>0.000274 (0.0005)</td>
<td>−2.5E-05 (−3.9E-05)</td>
</tr>
<tr>
<td>Manufacturing project 2</td>
<td>0.04 (0.06)</td>
<td>0.06197 (0.008764)</td>
<td>0.001732 (0.002739)</td>
<td>−0.00016 (−0.00021)</td>
<td>−0.00031 (−0.00039)</td>
</tr>
<tr>
<td>R&amp;D project</td>
<td>0.15 (0.2)</td>
<td>0.03354 (0.005)</td>
<td>0.003354 (0.005)</td>
<td>−0.00027 (−0.00077)</td>
<td>0.00025 (0.0003)</td>
</tr>
<tr>
<td>Index fund</td>
<td></td>
<td></td>
<td>0.03 (0.05)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>T-bill</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Note that the R&D project is a long-term project. Since we use a single table (Table 1) to describe time 0 assessments for both period 1 and period 2, we include the R&D project here. However, the R&D project only generates return in period 2 and it is the two-period return.*
decision processes to select them in alignment with the strategic objective that does not tend to change over the planning horizon (e.g., target credit rating). For simplicity, we also assume hazard risk exposure and other parameters remain constant over periods in stage 1. Additionally, we assume that the same set of parameter values are employed (for period 2) in stage 2, except for the insurance loading \( d \), which is assumed to increase in the recession period.9

In selecting the parameters for Table 2, we used real data, experience, and reasonable assumptions. When a firm internalizes our framework to create its own model, these parameters will be managerial inputs. We assume that the firm has a target credit rating of A (Moody’s), corresponding to a default probability of 0.08 percent, which is set to be the overall risk appetite.10 We also assume that downgrading to Baa represents the financial distress threshold, leading to about a 5 percent financial distress probability.11 In our baseline setting, all individual risk appetite parameters are currently chosen to be 5 percent, that is, no risk prioritization in the baseline setting.

Since there is hardly any publicly available data on operational losses for general industries, the operational risk loading factors \( \gamma_P \) and \( \gamma_A \) are chosen relative to the guidelines provided in Basel II (cf. Basel II, 2004). Finally, for the numerical illustration we assume the required capital obligations for each period to be proportional to the total capital in each period, that is, \( c^{(0)} = cC^{(0)} \) for period 1 and \( c^{(1)} = cC^{(1)} \) for period 2, although they could alternatively be specified as fixed dollar amounts.

Under the simplifying multivariate Normal assumption, we solve the optimization problem in this example using the techniques outlined in Charnes and Cooper (1963). The optimal values under the baseline setting are given in column 2 in Table 3. From Table 3, we can see that the majority of the capital goes to the two manufacturing

---

### Table 2

<table>
<thead>
<tr>
<th>Parameter Value</th>
<th>Risk Appetite</th>
<th>Risk Limits</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \alpha_1 )</td>
<td>( \alpha_2 )</td>
</tr>
<tr>
<td>-----------------</td>
<td>---------------</td>
<td>-------------</td>
</tr>
<tr>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Hazard Risk</th>
<th>Op Risk Factors</th>
<th>Strategic Factor</th>
<th>Borrowed Capital</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu )</td>
<td>( \sigma )</td>
<td>( d )</td>
<td>( \bar{d}^{(2)} )</td>
</tr>
<tr>
<td>0.01</td>
<td>0.1</td>
<td>0.2</td>
<td>0.3</td>
</tr>
<tr>
<td></td>
<td>( \gamma_P )</td>
<td>( \gamma_A )</td>
<td>( \gamma_5 )</td>
</tr>
<tr>
<td></td>
<td>0.2</td>
<td>0.1</td>
<td>0.5</td>
</tr>
</tbody>
</table>

---

9However, note that despite our simplifying assumptions, our model allows the choice of parameter values to differ for different periods or stages.


11Similar to Nocco and Stulz (2006), we use bond rating as a criterion for financial distress. If Baa is the financial distress threshold, the financial distress probability is the sum of the probability of downgrading from A to Baa and below. These probabilities are calculated based on Moody’s transition matrix.
Table 3
Optimal Operational Decisions for Each Stage

<table>
<thead>
<tr>
<th>Decision Variable</th>
<th>Baseline Optimal Decision (Period 2 Technology Change Not Anticipated at Time 0)</th>
<th>New Optimal Decision (Period 2 Technology Change Anticipated at Time 0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stage 1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Investment in manufacturing project 1 in period 1</td>
<td>0.5801</td>
<td>0.6223</td>
</tr>
<tr>
<td>Investment in manufacturing project 2 in period 1</td>
<td>0.1237</td>
<td>0.0917</td>
</tr>
<tr>
<td>Investment in R&amp;D project in period 1</td>
<td>0.0181</td>
<td>0.0217</td>
</tr>
<tr>
<td>Investment in the index fund in period 1</td>
<td>0.0466</td>
<td>0.0442</td>
</tr>
<tr>
<td>Investment in the Treasury bill in period 1</td>
<td>0.2190</td>
<td>0.2077</td>
</tr>
<tr>
<td>Proportion of hazard risk insured in period 1</td>
<td>0.9479</td>
<td>0.9479</td>
</tr>
<tr>
<td>Investment in manufacturing project 1 in period 2</td>
<td>0.2409</td>
<td>0.4169</td>
</tr>
<tr>
<td>Investment in manufacturing project 2 in period 2</td>
<td>0.3363</td>
<td>0.2386</td>
</tr>
<tr>
<td>Investment in the index fund in period 2</td>
<td>0.0720</td>
<td>0.0583</td>
</tr>
<tr>
<td>Investment in the Treasury bill in period 2</td>
<td>0.3383</td>
<td>0.2738</td>
</tr>
<tr>
<td>Proportion of hazard risk insured in period 2</td>
<td>0.9479</td>
<td>0.9479</td>
</tr>
<tr>
<td>Optimal return (for the planning horizon)</td>
<td>1.1797</td>
<td>1.1418</td>
</tr>
<tr>
<td>Stage 2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Investment in manufacturing project 1 in period 2</td>
<td>0.4996</td>
<td>0.4600</td>
</tr>
<tr>
<td>Investment in manufacturing project 2 in period 2</td>
<td>0.1682</td>
<td>0.1853</td>
</tr>
<tr>
<td>Investment in the index fund in period 2</td>
<td>0.0365</td>
<td>0.0391</td>
</tr>
<tr>
<td>Investment in the Treasury bill in period 2</td>
<td>0.2822</td>
<td>0.3021</td>
</tr>
<tr>
<td>Proportion of hazard risk insured in period 2</td>
<td>0.9479</td>
<td>0.9479</td>
</tr>
<tr>
<td>Optimal return (for the planning horizon)</td>
<td>1.1356</td>
<td>1.1343</td>
</tr>
</tbody>
</table>

projects and the Treasury bill due to the risk constraints. The firm revises its decisions toward the safer choices for period 2 in stage 2 in light of an updated (now assessed as recessionary) information set at time 1. Optimally the firm insures about 95 percent of hazard risk. Optimal expected return for the horizon is 18 percent at time 0 and reduces to 13.6 percent in stage 2 after recessionary information is incorporated.

Demonstration of the Dynamic Nature of Our Modeling Approach

In our dynamic ERM framework, the firm initially makes plans for both periods at time 0 in stage 1, taking into account investment opportunities and risks for both periods. While keeping all stage 2 assessments and all other assessments for both periods in stage 1 the same as in the baseline setting, we now illustrate the dynamics of the model by investigating how period 1 decisions would have changed if the firm anticipated at time 0 less favorable investment opportunities for manufacturing projects in period 2 (e.g., due to technology change) (note that in the baseline setting, time 0 assessment of these investment opportunities are assumed to be the same for both periods). Table 4 presents the description of this new set of investment opportunities for period 2 as assessed at time 0. Because of the interactivity of risks, stage 1 decisions for both periods will change, and since stage 2 decisions are based
on outcomes of period 1 decisions, they change as well. The optimal results under this new setting are shown in Table 3 (column 3).

From the side-by-side comparison of optimal decisions under the two settings in Table 3, we see that with the new time 0 assessment of period 2 investment opportunities, the firm invests more in the long-term project, which is more attractive now since the returns from manufacturing projects have worsen. The firm also becomes more conservative when planning for period 2. This interaction of the two periods manifests the dynamic nature of our framework.

### Comparative Impacts of Different Parameterizations

We now assess the impact of the decision parameters. Such parameters are not arbitrarily determined but firm-specific strategic choices. In doing these analyses, we explore how the firm might incorporate alternative considerations (e.g., risk prioritization), and examine the implications of parameter choice on ERM decisions. To conserve space we only present a reanalysis of stage 2 of the model. Table 5 presents a summary of the impact of each decision parameter as detailed in the rest of this section. In general, analyses below have illustrated that our formulation leads to intuitive results including how the strategic decisions reflect risk considerations, the interaction among different risk categories in an ERM setting, the increased efficiency of holistic decision making rather than that in “silos,” the important role of risk management in achieving strategic goals, etc. These help demonstrate the operationalizability of our ERM framework.
# Table 5

Optimal Stage 2 Decisions With Varied Parameter Value Choices Under Alternative Considerations

<table>
<thead>
<tr>
<th>Panel A: Alternative Choices of Risk Appetite Parameters</th>
<th>Baseline</th>
<th>Manu Project 1</th>
<th>Manu Project 2</th>
<th>Index Fund</th>
<th>T-Bill</th>
<th>Insurance</th>
<th>Optimal Return</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) Appetite I: High project, Low hazard&lt;sup&gt;a&lt;/sup&gt;</td>
<td>0.4996</td>
<td>0.2698</td>
<td>0.0461</td>
<td>0.3558</td>
<td>0.9625</td>
<td>1.1468</td>
<td></td>
</tr>
<tr>
<td>(2) Appetite II: Low financial, Low hazard&lt;sup&gt;b&lt;/sup&gt;</td>
<td>0.5009</td>
<td>0.1683</td>
<td>0.0163</td>
<td>0.3006</td>
<td>0.9625</td>
<td>1.1345</td>
<td></td>
</tr>
<tr>
<td>(3) Appetite III: High hazard&lt;sup&gt;c&lt;/sup&gt;</td>
<td>0.4991</td>
<td>0.1681</td>
<td>0.0367</td>
<td>0.2832</td>
<td>0.9035</td>
<td>1.1364</td>
<td></td>
</tr>
<tr>
<td>(4) Appetite IV: Low operational, Low hazard&lt;sup&gt;d&lt;/sup&gt;</td>
<td>0.4523</td>
<td>0.1625</td>
<td>0.0426</td>
<td>0.3288</td>
<td>0.9625</td>
<td>1.1337</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel B: Alternative Choices of Risk Limit Parameters</th>
<th>Baseline</th>
<th>Manu Project 1</th>
<th>Manu Project 2</th>
<th>Index Fund</th>
<th>T-Bill</th>
<th>Insurance</th>
<th>Optimal Return</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) Limit I: Low project&lt;sup&gt;e&lt;/sup&gt;</td>
<td>0.1476</td>
<td>0.0609</td>
<td>0.0892</td>
<td>0.6888</td>
<td>0.9479</td>
<td>1.1146</td>
<td></td>
</tr>
<tr>
<td>(2) Limit II: High operational&lt;sup&gt;f&lt;/sup&gt;</td>
<td>0.7787</td>
<td>0.1964</td>
<td>0.0013</td>
<td>0.0100</td>
<td>0.9479</td>
<td>1.1441</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel C: Alternative Choices of Other Parameters</th>
<th>Baseline</th>
<th>Manu Project 1</th>
<th>Manu Project 2</th>
<th>Index Fund</th>
<th>T-Bill</th>
<th>Insurance</th>
<th>Optimal Return</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) Other I: Low OP factors&lt;sup&gt;g&lt;/sup&gt;</td>
<td>0.7891</td>
<td>0.1973</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.9479</td>
<td>1.1444</td>
</tr>
<tr>
<td>(2) Other II: Low OP factors and Tight solvency&lt;sup&gt;h&lt;/sup&gt;</td>
<td>0.4115</td>
<td>0.0885</td>
<td>0.0291</td>
<td>0.4573</td>
<td>0.9496</td>
<td>1.1217</td>
<td></td>
</tr>
<tr>
<td>(3) Other III: High strategic&lt;sup&gt;i&lt;/sup&gt;</td>
<td>0.7389</td>
<td>0.0111</td>
<td>0.0201</td>
<td>0.2164</td>
<td>0.9479</td>
<td>1.1168</td>
<td></td>
</tr>
</tbody>
</table>

*Note:* All parameters are the same as in the baseline setting except as noted below:

<sup>a</sup> $\alpha_1 = 0.1, \alpha_4 = 0.01$; <sup>b</sup> $\alpha_2 = 0.01, \alpha_4 = 0.01$; <sup>c</sup> $\alpha_4 = 0.2, \alpha_3 = 0.01, \alpha_4 = 0.01$; <sup>d</sup> $\alpha_3 = 0.01, \alpha_4 = 0.01$; <sup>e</sup> $r_{p0} = 0.05$; <sup>f</sup> $\gamma_P = 0.1, \gamma_A = 0.05$; <sup>g</sup> $\gamma_P = 0.1, \gamma_A = 0.05, c = 0.88$; <sup>i</sup> $\gamma_S = 0.75$. 
The Impact of Different Risk Prioritizations. Under our formulation, risk prioritization is obtained by varying the individual risk appetite parameters (a larger $\alpha$ for low-priority risks). We discuss its impact on optimal operational decisions below.

1. **Risk appetite I: High appetite for project risk, prioritize hazard risk ($\alpha_1 = 0.1$ and $\alpha_4 = 0.01$)**

   A firm may have a higher risk appetite than the baseline for project risk since project risk is its core competence, and taking necessary risk yields competitive advantage. Also, if the firm does not have expertise in managing hazard risk, it may want to prioritize hazard risk (i.e., a lower risk appetite), especially since this risk can be better managed through insurance. From Table 5, we see that the result of these changed risk appetites is a 61 percent increase in the optimal investment in the riskier manufacturing project 2. By requiring tighter control of the remaining hazard risk (i.e., lower $\alpha_4$), a larger proportion is insured. The total expected return also increases.

2. **Risk appetite II: Prioritize financial risk and hazard risk ($\alpha_2 = 0.01$ and $\alpha_4 = 0.01$)**

   In addition to hazard risk, a general industrial firm may find financial risk less attractive. From Table 5 we see that adjusting the risk appetite parameters to provide for smaller financial risk appetite leads to less investment in the “riskier” index fund and a lower expected total return.

3. **Risk appetite III: High appetite (low priority) for hazard risk ($\alpha_4 = 0.2$)**

   A larger firm may have a relatively high-risk appetite for hazard risk and choose to retain a higher proportion of it as insurance is costly. Savings on insurance loadings allow more capital to be invested, and hence this trade-off leads to higher optimal expected returns. The implication is that maximally insuring hazard risk may not be the best strategy, especially when the firm foresees good alternative investment opportunities, and/or has a higher capacity for risk retention.

4. **Risk appetite IV: Prioritize operational risk and hazard risk ($\alpha_3 = 0.01$ and $\alpha_4 = 0.01$)**

   We show that the recognition of operational risk is important in a firm’s operating decisions. Higher prioritization (or lower appetite) can arise because operational risk is less well understood. While the firm maintains the same appetite for project risk, investment in both projects are reduced, since these real projects have larger exposure to operational risk than financial investments ($\gamma_P > \gamma_A$). Operational risk highlights the interactions among different risk categories as is characteristic of ERM.

Risk Limits. Now we discuss the effects of changing risk thresholds or limits. Risk limits can be set based on the assessment of each risk’s impact on financial distress or can be derived from managerial assessment of historical data and operating experience.

1. **Risk limit I: Lower risk limit for project risk ($r_{P0} = 0.05$)**
A lower permissible risk limit (higher hurdle rate) results in greatly reduced investment in both projects. Within the context of this scenario, it is important to note that the previous strategic constraint that “investment in projects must be at least 50 percent of the capital” must be loosened in order to find a feasible solution for the ERM framework in this setting. This does not necessarily mean that the firm cannot use individual risk limits but rather shows that all parameters need to be simultaneously carefully chosen. After adjusting the strategic constraint, we find that more capital goes to financial assets, and the total expected return is reduced.

(2) Risk limit II: Higher risk limit for operational risk ($l_{op} = 0.3$)

Similar to having a high-risk appetite for operational risk, relaxing limits for operational risk results in more capital invested in projects because the high project loading on operational risk had previously limited investment in projects.

(3) Risk limit III: Lower risk limit for financial risk ($r_{A0} = 0.05$)

When the risk limit for financial risk is set at $r_{A0} = 0.05$, no feasible solution exists. In this case (as previously), the decision parameters need to be reassessed by managers, or the firm may need to search for alternative investment opportunities if they are available. This highlights an ERM benefit—if managed in silos, individual decisions may lead (through interactivity with other risks) to decisions incompatible with global corporate strategic goals; however, infeasibility in the ERM model can highlight such incompatibilities and bring those to the attention of the decision makers.

Other Parameters

(1) Other factor I: Low operational risk factors ($\gamma_P = 0.1$ and $\gamma_A = 0.05$)

The parameters $\gamma_P$ and $\gamma_A$ form the basis of our quantification of operational risk. The factors developed from Basel Capital Accord II can be used as a guideline; however, in practice they could be made firm specific. The presented operational risk loadings $\gamma_P$ and $\gamma_A$ could also be lowered by undertaking risk management actions to reduce operational risk exposure (e.g., refining organizational supervisory structure, employee training, etc.). This in turn results in all resources going to the more profitable real project investments and thus increases the total expected return.

(2) Other factor II: Low operational risk factors ($\gamma_P = 0.1$ and $\gamma_A = 0.05$), tight solvency constraint ($c = 0.88$)

Operational risk factors are important in the presence of a tighter solvency constraint. If the firm raises the proportion of borrowed capital $c$ to, for example, 0.88, no feasible solution can be found within the original baseline parameterization ($\gamma_P = 0.2$ and $\gamma_A = 0.1$). However, if one could lower these two factors to 0.1 and 0.05 (e.g., through
risk management), then the solvency risk constraint can be satisfied.\textsuperscript{12} This finding has important risk management implications: risk management is an important tool for managers to reconcile potentially conflicting risk considerations and is critical in situations where strategic objectives cannot be achieved without risk management support (e.g., in a heavily leveraged firm).

(3) \textit{Other factor III: High strategic or regulation parameter ($\gamma_S = 0.75$)}

The parameter $\gamma_S$ reflects the firm’s strategic consideration (e.g., to retain a competitive market presence), or it may be a response to regulatory requirements (e.g., a firm may only be allowed to invest a certain proportion of capital in certain assets or an insurance firm can be prohibited from exiting a market). Here we find more capital is invested in the “safer” project and the optimal return drops significantly.

(4) \textit{Other factor IV: Insurance loading ($d$)}

When a solvency constraint is not imposed, the magnitude of the loading $d$ does not affect the proportion of hazard risk optimally insured; however, if the solvency constraint is added, the amount of hazard risk insured may change.

In summary, note that the maximum total expected return is obtained by \textit{simultaneously} considering all constraints integrated in the ERM framework. If risks were handled in silos, the decisions may fail to honor one or more risk constraints and thus may not be optimized to achieve the goals of the firm.

\section*{Conclusion}

This article fills a gap in the ERM literature by presenting a single quantitative model for operationalizing ERM decision making. Using an optimization approach, we introduce a mathematically formulated dynamic framework that allows firms to align risk appetite for multiple projects, financial investments, and operational and uninsured hazard exposure with their overall strategic goals over time. More specifically, the model optimizes these strategic goals while constraining risks via the delineation of acceptable risk levels allowing for various risk appetites, similar in spirit to the traditional two-dimensional risk analysis approach familiar to decision makers. The commonly used VaR risk measure is adopted to formulate risk constraints so as to be able to incorporate fundamental characteristics of ERM, such as risk appetite and risk prioritization. Our approach to ERM operationalization is dynamic in that it depicts the firm as first making strategic plans for the entire multiperiod planning horizon and updating its operational decisions every period as new information becomes available. This is important to the development of ERM in general as firms are always making decisions in a nonstationary environment.

The model is illustrated with a numerical implementation that shows how a hypothetical firm can make optimal capital allocation decisions while managing across business units, risk types, and decision parameters (e.g., risk appetite and risk prioritization). We have also provided discussions of managerial choice of decision

\textsuperscript{12}Numerical experiments suggest adjusting these two factors is an easy way to satisfy stringent overall constraints compared to adjustment of other parameters.
parameters and its implications for the ERM operational decisions. With these, the article provides implementation guidance to firms seeking to operationalize ERM.

While we provide a formulation that uses the frequently employed VaR measure, other risk measures, such as variance, semivariance, and expected shortfall, can be used in place of VaR to formulate the constraint set and still yield a computationally feasible optimization problem within our model. The different solution techniques entailed by various model specifications are explicitly prescribed in the article.\textsuperscript{13}

This article also makes a contribution to optimal capital allocation/portfolio selection literature under VaR constraints (cf. Campbell, Huisman, and Koedijk 2001; Danielsson et al., 2008). We could find no paper in this literature that addresses optimal allocation when several VaR constraints are imposed at the individual levels, as well as at the aggregate level, as is done in our article. We go beyond portfolio selection models (such as Markowitz, 1952), as the article considers multiple risk types, is dynamic, uses a different risk metric formulation, allows a choice of desirable risk levels and risk appetite, incorporates constraints, and has the ability to treat different types of risks differently. The combination of these characteristics in our formulation is unique in the literature.

While our formulation is a contribution to ERM operationalization and related literature, it underscores numerous opportunities for future research. Possible extensions to our formulation include (1) other implementation of operational risk, (2) forming constraints based on aggregated distributions derived from empirical (or simulated) marginal distributions using a copula model for dependence (cf. Rosenberg and Shuermann, 2006), (3) adding/modifying risk types and measures, (4) explicitly incorporating risk management strategies for all risk types, (5) switching to other business objectives rather than maximizing end-of-horizon wealth, and (6) different themes and processes for the choice of decision parameters. Our formulation is sufficiently flexible to be adapted to other strategic considerations of interest to decision makers.

\textbf{Appendix}

Period 2 constraints resemble those for period 1 (e.g., Cons 2-1 corresponds to Cons 1-1), except for the treatment of long-term investment components (capital committed in period 1 and returns generated in period 2) in the relevant risk constraints. Superscripts are also changed from “(1)” to “(2)” consequently to reflect the time periods. They are explicitly shown below.

\textit{Constraints for period 2 in stage 1:}

Using $\tilde{C}^{(1)} = C^{(0)}(w^{(1)}_S r^{(1)}_S - u^{(1)}d^{(1)} \mu^{(1)})$ and $\tilde{c}^{(1)} = E[\tilde{C}^{(1)} | J_o] = C^{(0)} E (w^{(1)}_S r^{(1)}_S - u^{(1)}d^{(1)} \mu^{(1)})$, we have

\textsuperscript{13}Prekopa (1971), Cooper, Lelas, and Sullivan (2006), and Poojari and Vargheses (2008) provide guidance relevant to the computations involved in using other types of risk measures in our formulation.
\[ P \left[ \hat{C}^{(1)} P_{SP} (2) + C^{(0)} P_{LP} r_{LP} \leq \hat{C}^{(1)} P_{SP} e_{r_{SP}}^{(2)} \right] \leq \alpha_1, \quad \text{(Cons 2-1)} \]

\[ P \left[ \hat{C}^{(1)} P_{SA} (2) + C^{(0)} P_{LA} r_{LA} \leq \hat{C}^{(1)} P_{SA} e_{r_{SA}}^{(2)} \right] \leq \alpha_2, \quad \text{(Cons 2-2)} \]

\[ P \left[ \gamma_P^{(2)} \hat{C}^{(1)} P_{SP} (2) + \gamma_P^{(2)} C^{(0)} P_{LP} r_{LP} + \gamma_A^{(2)} \hat{C}^{(1)} P_{SA} (2) + \gamma_A^{(2)} C^{(0)} P_{LA} r_{LA} \geq \hat{C}^{(1)} l_{op}^{(2)} \right] \leq \alpha_3, \quad \text{(Cons 2-3)} \]

\[ P \left[ (1 - u^{(2)}) h^{(2)} \hat{C}^{(1)} \geq h^{(2)} e^{(1)} \right] \leq \alpha_4, \quad \text{(Cons 2-4)} \]

\[ P \left[ \gamma_P^{(2)} \hat{C}^{(1)} P_{SP} (2) + \gamma_P^{(2)} C^{(0)} P_{LP} r_{LP} + \gamma_A^{(2)} \hat{C}^{(1)} P_{SA} (2) + \gamma_A^{(2)} C^{(0)} P_{LA} r_{LA} \geq \hat{C}^{(1)} l_{op}^{(2)} \right] \leq \alpha_5, \quad \text{(Cons 2'-3)} \]

\[
\begin{align*}
 w^{(2)}_{SP} e + w^{(2)}_{SA} e + u^{(2)} (1 + d^{(2)}) \mu^{(2)} & \leq 1; \ w^{(2)}_{SP} e \geq \gamma_S^{(2)}; \ w^{(2)}_{SP} \geq 0, \ w^{(2)}_{SA} \geq 0, \ 0 \leq u^{(2)} \leq 1. \\
 & \quad \text{(Cons 2-6, 7, 8)}
\end{align*}
\]

Stage 2 constraints (for period 2) are similar to period 2 constraints in stage 1 (e.g., Cons 2'-1 corresponds to Cons 2-1), except expected capital \( \hat{C}^{(1)} \) is now replaced by actual capital \( C^{(1)} \) and updated information \( I_1 \) is used to form probability distributions. A “bar” (\( \bar{\cdot} \)) is used to indicate updated information \( I_1 \) and updated decisions. They are explicitly shown below.

**Constraints for period 2 in stage 2:**

Using \( C^{(1)} = C^{(0)} [w^{(2)}_{SP} e_S^{(1)} - u^{(1)} s d^{(1)} \mu^{(1)}] \), we have

\[ P \left[ C^{(1)} P_{SP} (2) + C^{(0)} P_{LP} r_{LP} \leq C^{(1)} P_{SP} e_{r_{SP}}^{(2)} \right] \leq \alpha_1, \quad \text{(Cons 2'-1)} \]

\[ P \left[ C^{(1)} P_{SA} (2) + C^{(0)} P_{LA} r_{LA} \leq C^{(1)} P_{SA} e_{r_{SA}}^{(2)} \right] \leq \alpha_2, \quad \text{(Cons 2'-2)} \]

\[ P \left[ \gamma_P^{(2)} C^{(1)} P_{SP} (2) + \gamma_P^{(2)} C^{(0)} P_{LP} r_{LP} + \gamma_A^{(2)} C^{(1)} P_{SA} (2) + \gamma_A^{(2)} C^{(0)} P_{LA} r_{LA} \leq C^{(1)} l_{op}^{(2)} \right] \leq \alpha_3, \quad \text{(Cons 2'-3)} \]

\[ P \left[ (1 - \bar{u}^{(2)}) \bar{h}^{(2)} C^{(1)} \geq \bar{h}^{(2)} C^{(1)} \right] \leq \alpha_4, \quad \text{(Cons 2'-4)} \]
\begin{align*}
\mathbf{P} \begin{bmatrix}
(1 - \tilde{\gamma}_p^{(2)})C(1) \tilde{w}_{SP}^{(2)} + (1 - \tilde{\gamma}_p^{(2)})C(0) \tilde{w}_*^{*L}LP + (1 - \tilde{\gamma}_A^{(2)})C(1) \tilde{w}_{SA}^{(2)} \\
+ (1 - \tilde{\gamma}_A^{(2)})C(0) \tilde{w}_*^{*LA}LA - (1 - \tilde{u}^{(2)})\tilde{h}^{(2)}C(1) - \tilde{u}^{(2)}(1 + \tilde{d}^{(2)})\tilde{\mu}^{(2)}C(1) \leq c^{(1)}
\end{bmatrix} \leq \alpha_5,
\end{align*}
(Cons 2'-5)
\begin{align*}
\tilde{w}_{SP}^{(2)} + \tilde{w}_{SA}^{(2)} + \tilde{u}^{(2)}(1 + \tilde{d}^{(2)})\tilde{\mu}^{(2)} \leq 1; \\
\tilde{w}_{SP}^{(2)} \geq \tilde{\gamma}_S^{(2)}; \\
\tilde{w}_{SA}^{(2)} \geq 0, \\
0 \leq \tilde{u}^{(2)} \leq 1.
\end{align*}
(Cons 2'-6,7,8)

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